4. (30 points)   Consider a pure-exchange economy in which there are two goods and two consumers. The total endowment is one unit of each good. The utility function of Consumer 1 is

$$u_1(c_{11}, c_{12}) = \min\{c_{11}, 2c_{12}\},$$

where c_{1k} is the consumption of good k by Consumer 1 ($k = 1, 2$). Similarly, the utility function of Consumer 2 is

$$u_2(c_{21}, c_{22}) = \min\{2c_{21}, c_{22}\}$$

Let $p = (p_1, p_2) \geq 0$ denote prices.

(a) Find the set of Pareto-efficient allocations for the economy and illustrate them in an Edgeworth Box.

(b) Find all competitive equilibrium prices in the economy when Consumer 1 has the initial endowment $(1, 0)$ and Consumer 2 has the initial endowment $(0, 1)$.

(c) Describe the core of the economy of part (b) and compare it to that of the four-agent replica economy (with two agents having preferences and endowments identical to the original Consumer 1 and two other agents having preferences and endowments identical to the original Consumer 2). For this part it is sufficient to write down a system of inequalities that describes the core (you need not find a simple description of the set defined by the inequalities).

(d) Define the equal-treatment property of a replica economy. Will the core of the economy in part (c) satisfy the equal-treatment property? Explain.